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Atmospheric structure of K *dwarfs*

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Abstract

A representative model atmosphere is presented for a *K* dwarf with an effective temperature of 4000° and a log surface gravity of 4.5. The model is non-gray and radiative flux constancy is enforced to within 1 per cent. Ten sources of opacity and scattering are included, but water vapor opacity is omitted.

I. INTRODUCTION

DURING THE PAST FEW YEARS there has been a growing interest in cool stars as is evidenced in the colloquium edited by M. Hack (1967). Kumar (1964) and Gingerich, Latham, Linsky, and Kumar (1967) began the work of computing accurate non-gray and flux-constant radiative models for late-type stars. During the past year a computer program for model atmospheres has been written and tested at the University of Virginia.

This paper presents a representative model of a late dwarf in radiative equilibrium. The radiative atmosphere computation is one aspect of a general investigation of radiative, convective and scaled solar atmospheres for dwarfs. Twelve strong-line profiles have been computed for each one of a grid of model atmospheres for comparison with observation. Recently several authors have published one or more profiles derived from high dispersion plates for each of 4 early *K* dwarfs and one *M*2 dwarf. We have acquired a few additional profiles at the Kitt Peak National Observatory. The differences between the various atmospheres and the resulting profiles are being discussed and compared with available observations (Hershey 1969).

The model atmosphere presented here has an effective temperature of 4000° and $\log g = 4.5$. This corresponds closely to generally accepted values for a *K*5 dwarf (Allen 1963). The composition is the same as given by Goldberg, Müller, and Aller (1960), with the exception that the He/H ratio was taken to be 0.1 by number instead of 0.2. A total of thirty elements are included. A list of these elements and abundances may be found in the paper of Gingerich *et al.* (1967).

The equation of state procedure includes five states of hydrogen, H, H^+, H^-, H_2^+ and H_2. Only first ionizations have been included for the 28 metals. No molecules other than H_2 have been included; in particular, water vapor has not been included in the model. The equilibrium functions and the method for solving the equation of state are taken from Mihalas (1967).

The method of numerical integration of the differential equation of hydrostatic equilibrium, $dP/d\tau_0 = g/\kappa_0$, is the same as used by Gingerich *et al.* and is described by Ralston and Wilf (1960). The reference wavelength adopted was 10000 Å which lies much nearer to the wavelength of maximum surface flux than do wavelengths in the visible region.

The initial pressure was found by a method similar to that described by 8
Mihalas (1967). This method assumes an electron pressure by trial and error which corresponds to a point well above the first optical depth of the model atmosphere and then integrates down to the first point. This in effect starts the integration above the first point in the table and provides a self-consistent-starting value for the solution of the differential equation.

II. OPACITIES

The opacities given here are not listed in order of importance. The importance of individual opacities often varies strongly with wavelength and depth in an atmosphere. The behavior of the opacities at $\tau_0 = 1$ can be seen in Figure 1.

1. Hydrogen. The bound–free and free–free absorption are the well known expressions given by Menzel and Pekeris (1936). Hydrogen is an insignificant opacity source in cool stars.

2. Negative Hydrogen Ion. This ion is the dominant opacity source in *K* dwarfs. Polynomials given by Gingerich *et al.* (1964) for the bound-free and free–free components have been used.

3. Positive Molecular Hydrogen Ion. The expression used includes a polynomial fit to the tables given by Mihalas (1967).

4. Negative Molecular Hydrogen Ion. No bound state is known to exist. Free–free opacity due to H_2^- is important, but no accurate cross sections are available. A recent suggestion by Vardya (1966) has been followed, which is to set the H_2^- cross section per H_2 molecule equal to twice the cross section of H_{ff}^- per H atom.

5. Rayleigh Scattering from Hydrogen and H_2. Gingerich (1964) has given a simple expression for the hydrogen scattering cross section per neutral H

atom. Mihalas (1967) has given an expression for the scattering cross section of the H_2 molecule.

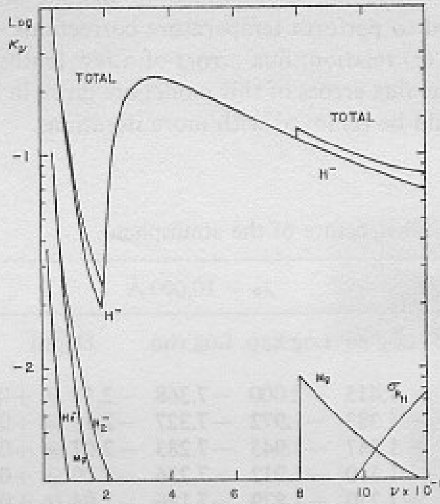

FIGURE 1 The monochromatic opacities at $\tau_{10,000} = 1$; at this depth $T = 3998°K$, $\log P = 5.525$ and $\log P_e = 0.396$.

6. Electron Scattering. This is completely negligible, being about 10^{-5} of total opacity in late-type dwarfs, but it was included as a matter of course.

7. Negative Helium Ion. The opacity computation of He^- interpolates in tables by McDowell *et al.* (1966). A computer library program was used to fit the table and produce the opacity as a function of θ and λ.

8. Magnesium and Silicon. Crude opacity expressions were formed following Unsöld (1955) and several absorption edges are included. The notable feature of the metal opacities is to block the small amount of black-body flux bluer than 2515 Å.

III. SOLUTION OF THE INTEGRAL EQUATION FOR THE SOURCE FUNCTION

Although scattering is not strong in any part of the model, the source function does differ from $B_\nu(T)$ in the visible region of the spectrum by 5 to 10 per cent at the surface. In the ultraviolet the differences are larger but very little flux is carried there. The integral equation for the source function was solved following the method given by Mihalas (1967). This is an iterative method which finds successively better values of $(J_\nu - B_\nu)$. The solution was checked by substituting S_ν into the integral equation independently. The error of the solution is well under 1 part in 1000.

IV. FLUX CONSTANCY

The Avrett–Krook procedure as modified to include scattering (Mihalas 1967), has been used to perform temperature corrections. In four iterations from an assumed $T(\tau)$ relation, flux errors of a few tenths of a per cent are usually achieved. The flux errors of this model are given in Table I. All of the remaining error could be removed with more iterations.

TABLE I The physical structure of the atmosphere.

$T_e = 4000°$ $\lambda_0 = 10{,}000$ Å $\log g = 4.5$

Tau	Temp	Log p	Log pe	Log kap	Log rho	Depth	Conv.	Flux error
0.001	3241	3.923	−1.415	−2.000	−7.368	−2.18 @ +07		0.0%
0.001	3242	3.963	−1.383	−1.972	−7.327	−2.13 @ +07		0.0%
0.002	3243	4.005	−1.347	−1.943	−7.283	−2.07 @ +07		0.0%
0.002	3244	4.049	−1.310	−1.912	−7.236	−2.00 @ +07		0.0%
0.003	3246	4.096	−1.270	−1.879	−7.186	−1.94 @ +07		0.0%
0.003	3247	4.146	−1.228	−1.845	−7.134	−1.87 @ +07		0.0%
0.004	3249	4.197	−1.184	−1.810	−7.080	−1.80 @ +07		0.0%
0.005	3251	4.249	−1.138	−1.775	−7.025	−1.73 @ +07		0.0%
0.006	3254	4.303	−1.091	−1.739	−6.968	−1.65 @ +07		0.0%
0.008	3257	4.358	−1.042	−1.702	−6.909	−1.58 @ +07		0.0%
0.010	3260	4.414	−0.993	−1.665	−6.850	−1.50 @ +07		0.0%
0.013	3265	4.471	−0.941	−1.628	−6.789	−1.43 @ +07		0.0%
0.016	3270	4.528	−0.889	−1.590	−6.729	−1.35 @ +07		0.0%
0.020	3277	4.586	−0.836	−1.552	−6.667	−1.28 @ +07		0.0%
0.025	3284	4.645	−0.781	−1.513	−6.606	−1.20 @ +07		0.0%
0.032	3294	4.704	−0.725	−1.474	−6.544	−1.13 @ +07		0.0%
0.040	3305	4.763	−0.668	−1.435	−6.483	−1.05 @ +07		0.0%
0.050	3319	4.822	−0.610	−1.394	−6.423	−9.74 @ +06		0.0%
0.063	3335	4.881	−0.551	−1.353	−6.363	−8.98 @ +06		0.0%
0.079	3354	4.940	−0.490	−1.310	−6.305	−8.23 @ +06		0.0%
0.100	3377	4.999	−0.427	−1.265	−6.248	−7.48 @ +06		0.0%
0.126	3405	5.057	−0.363	−1.218	−6.194	−6.73 @ +06		0.0%
0.158	3438	5.114	−0.296	−1.170	−6.142	−5.98 @ +06		0.0%
0.200	3476	5.171	−0.227	−1.119	−6.092	−5.24 @ +06	C	0.0%
0.251	3522	5.226	−0.156	−1.065	−6.046	−4.49 @ +06	C	0.0%
0.316	3575	5.280	−0.080	−1.008	−6.003	−3.74 @ +06	C	0.0%
0.398	3638	5.333	−0.001	−0.948	−5.964	−3.00 @ +06	C	0.1%
0.501	3710	5.384	0.085	−0.884	−5.929	−2.25 @ +06	C	0.1%
0.631	3793	5.433	0.178	−0.815	−5.898	−1.50 @ +06	C	0.1%
0.794	3888	5.480	0.282	−0.741	−5.871	−7.45 @ +05	C	0.1%

TABLE I—(*cont.*)

$T_e = 4000^\circ$ | $\lambda_0 = 10{,}000$ Å | $\log g = 4.5$

Tau	Temp	Log p	Log pe	Log kap	Log rho	Depth	Conv.	Flux error
1.000	3998	5.524	0.396	−0.661	−5.847	0.00 @ +00	C	0.1%
1.259	4122	5.566	0.523	−0.576	−5.826	7.37 @ +05	C	0.1%
1.585	4263	5.606	0.661	−0.487	−5.809	1.46 @ +06	C	0.1%
1.995	4423	5.643	0.807	−0.399	−5.794	2.18 @ +06	C	0.1%
2.512	4604	5.678	0.957	−0.315	−5.781	2.90 @ +06	C	0.1%
3.162	4807	5.712	1.105	−0.240	−5.769	3.63 @ +06	C	0.2%
3.981	5035	5.746	1.250	−0.174	−4.758	4.39 @ +06	C	0.2%
5.012	5289	5.780	1.395	−0.112	−5.747	5.20 @ +06	C	0.2%
6.310	5569	5.813	1.557	−0.036	−5.736	6.05 @ +06	C	0.3%
7.943	5877	5.845	1.761	0.081	−5.728	6.90 @ +06	C	0.3%
10.000	6212	5.872	2.017	0.251	−5.726	7.65 @ +06	C	0.3%
12.589	6576	5.893	2.310	0.457	−5.730	8.27 @ +06	C	0.2%
15.849	6973	5.908	2.617	0.680	−5.741	8.75 @ +06	C	0.4%
19.953	7406	5.919	2.929	0.911	−5.756	9.12 @ +06	C	−0.3%
25.119	7879	5.927	3.236	1.145	−5.776	9.40 @ +06	C	0.5%
31.623	8409	5.933	3.543	1.390	−5.799	9.62 @ +06	C	−0.1%
39.811	8980	5.937	3.836	1.643	−5.825	9.78 @ +06	C	−0.3%
50.119	9612	5.940	4.120	1.917	−5.855	9.90 @ +06	C	0.4%
63.096	10271	5.941	4.379	2.198	−5.888	9.98 @ +06	C	−0.8%
79.433	10962	5.943	4.615	2.485	−5.924	1.00 @ +07	C	0.3%

V. DISCUSSION

The essential parameters of the 4000° radiative model are given in Table I. The effective temperature is reached at almost exactly $\tau_0 = 1$. The gas pressure at $\tau_0 = 1$ is over twice its value in the sun at the same depth.

Using Schwarzschild's criterion, it is found that convection sets in at $\tau_0 = 0.2$ and thus the model cannot be very realistic for a K dwarf star. A flux constant radiative model is useful for comparison with convective and scaled solar models, for comparison with observation, and for testing the computer program.

Comparison of Figures 1 and 2 shows that the sharp H^- opacity minimum at 16,400 Å causes a sharp flux maximum which is accentuated by the peak of the black-body curve falling in that region. This was pointed out a few years ago by Kumar (1964) and Gingerich and Kumar (1964).

For the effective temperature of 4000° used here, the black-body peak on the frequency scale is much less sharp and is about 3000 Å further to the blue than the flux peak in Figure 2. The opacity minimum thus holds the flux peak at a constant wavelength over a range of effective temperatures. Observational tests of the reality of the theoretical H^- opacity minimum should be possible.

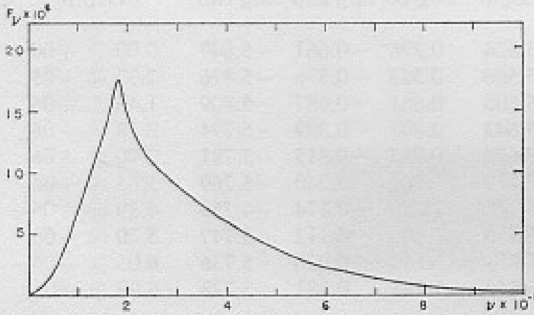

FIGURE 2 The monochromatic flux at the surface for $T_e = 4000°$ and $\log g = 4.5$.

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